Rural Investment and the Dynamic Cost of Income Uncertainty

T. Heikkinen
MTT Economic Research
Luutnantintie 13, 00410 Helsinki FINLAND
email:tiina.heikkinen@mtt.fi
and K. Pietola, MTT Economic Research

June 14, 2006

Abstract

This paper studies optimal investment and the dynamic cost of income uncertainty, applying a stochastic programming approach. The motivation is given by a case study in Finnish agriculture. Investment decision is modelled as a Markov decision process, extended to account for risk. A numerical model for computing the dynamic uncertainty cost is presented, applying the classical expected value of perfect information. The uncertainty cost depends on the volatility of income; e.g. with stationary income, the dynamic uncertainty cost is equivalent to a dynamic option value of postponing investment. In the case study, the investment decision is sensitive to risk. The model can be applied e.g. in planning investment subsidies for maintaining target investments.

Keywords: OR in agriculture, real options, investment analysis, stochastic programming

1 Introduction

Income variability is a persistent problem in agriculture. Currently, common Agricultural Policy (CAP) after EU enlargement implies many uncertainties regarding future agricultural income in Northern Europe. This paper addresses the cost of income uncertainty, focusing on a case study in Finnish agriculture. In the case study, due to a high degree of policy uncertainty, the rate of return on investment can be uncertain. Applying a stochastic programming approach [Birge and Louveaux1997, Prekopa1995], a model for dynamic uncertainty cost is presented, modifying the classical expected value of perfect information. The model is based on assuming a risk-neutral decision maker. Risk implies an additional cost of uncertainty. To study the effect of risk on investment, the model is extended to explicitly accounting for risk, based on the stochastic programming approach introduced in [Levitt and Ben-Israel2001] (previously with applications to inventory control and the maintenance problem).

Optimal investment is studied applying real investment options, see e.g. [Dixit and Pindyck1994, Keswani and Shackleton2006]. Investment options typically involve three parameters: the initial and accumulated costs, the flexibility in timing the investment and the uncertainty regarding the future rewards. [Dixit and Pindyck1994] studies the optimal investment decision as a Markov decision process (MDP) defined in continuous time (Ito process) and with a continuous state space. To simplify numerical analysis, this paper applies a discrete time MDP with discretized state space to study optimal investment. Mean reverting processes are frequently used in real option models; this paper assumes a mean reverting income process with a non-increasing expected value.

The main results can be summarized as follows:

- A numerical model for quantifying the dynamic uncertainty cost is presented, modifying the expected value of perfect information (EVPI) [Birge and Louveaux1997] to a dynamic setting.

- In the special case of stationary income, the proposed dynamic uncertainty cost is equivalent to a dynamic option value of postponing investment. In case study examples, the dynamic cost of income uncertainty is approximately 5% of the expected value of investment, exceeding 15 % in examples with higher income volatility.

- The dynamic investment decision is sensitive to risk.

It remains a topic for future work to obtain the subjective probability distributions [1], e.g. by conducting a survey similar to that in [Lagerkvist2005] using a visual impact method [Hardaker et al.1997]. Then the model can be applied e.g. in planning investment subsidies. The lack of complete information causes inefficiency by inducing under-investment, cf. [Lagerkvist2005]; a high uncertainty cost deteriorates the efficiency of investment subsidies. For an alternative empirical approach to measuring the cost of income uncertainty (risk representing the variance-covariance structure of firm's income), see e.g. [Amegbeto and Featherstone1992].

That risk matters to optimal investment is a result reached in this paper within a dynamic model. Related work [Alvarez and Stenbacka2004] applies an analytical continuous time model, whereas in this paper similar results are obtained via simulation in a discrete time setting. On the other hand, previous work discussed in [Lagerkvist2005] (with reference to [Knapp and Olson1996]) suggests risk aversion to be of less importance in a dynamic model than in a static setting.

Recently, related work in [Verkammen] has applied a discrete time MDP model to study optimal farmland investment assuming a price subsidy or a decoupled direct payment. The investment results, assuming a risk-neutral decision maker, are not very sensitive to the variability in the income. This is due to the assumed stationarity of the revenues. However, as emphasized in [Dixit and Pindyck1994], in general both the growth in the value of investment and uncertainty (as modelled by the variability in income) affect the optimal investment decision. Recent econometric evidence supports a nonlinear uncertainty-investment relation: for low levels of uncertainty an increase in uncertainty has a positive effect on investment, while for high levels of uncertainty an increase in uncertainty lowers investment [Bo and Lensink2005]. The investment model presented in this paper suggests that taking risk into account in general affects the uncertainty-investment relation.

[1] In some of the case examples the mean uncertainty cost is quite robust to changes in underlying probabilities.

2 Optimizing Investment

The notations are introduced in what follows, considering the investment decision of a representative firm in a discrete time model. Letting ρ denote the internal rate of return, denote the discount factor of the firm by b:

$$b = 1/(1+\rho) \in (0, 1].$$

Let r_t denote the return on investment (%) per time period at time t. The timing of the investment is considered as the decision variable. The flexibility in timing the investment affects the value of investment [Dixit and Pindyck1994, Keswani and Shackleton2006]. In this paper the decision-maker is assumed to have full flexibility in timing the investment. Denoting by I the total available budget for the investment at t, the decision I_t at t satisfies

$$I_t \in \{0, I\} \; \forall t. \tag{1}$$

The future value of investment is random, due to variability in the rate of return. Letting I_a denote the aggregate budget, the aggregate budget constraint requires:

$$\sum_{t=1}^{T} I_t \leq I_a. \tag{2}$$

The dynamic optimization problem can be written as:

$$\max \; E[\sum_{t=1}^{\infty} b^t (r_t + \sum_{k=t+1}^{\infty} b^k r_k) I_t - I_t] \tag{3}$$

where E denotes the expectation operator. Two versions of problem (3) subject to (2) are studied in what follows; in the first model, it is assumed that r_t is observable when the investment decision is made at time t; in the second model only r_{t-1} is observable at time t. To begin with it is assumed that the total available amount for investment can be spent at any time; later this simplification is removed by introducing period-specific budget constraints.

Since the future values of the investment are unknown, there is an opportunity cost to making the investment decision at $t = 1$ [Dixit and Pindyck1994]; the firm has the option to postpone the investment decision. In what follows a time-correlated income process is assumed, to study optimal investment under decreasing income expectations, allowing the firm to make the investment decision at any time. Assuming time-correlated income, the optimal investment rule will be threshold-based, with time-dependent thresholds.

3 Investment with Time-Correlated Income

In what follows the income process is assumed to be non-increasing, capturing decreasing expectations regarding income subsidies. The motivation is given by a case study from Finnish agriculture, summarized in what follows applying a discrete time Markov model. A stochastic programming model for measuring the uncertainty cost is introduced, based on two optimization models. In the first model (Model 1), the value of investment is observable at the time the investment decision is made; in the second model (Model 2), the value of investment is unobservable.

A Case Study

Milk production is the most important production line in Finnish agriculture [Lehtonen2004]. Table 1 summarizes the expected profitability of investment in milk production in Finland, based on [Uusitalo et al.2004]. For example, assuming the investment subsidy grows by 20 % (or by 50 %, depending on production unit) and assuming the producer price decreases by 15 % from 2003 level by 2007, the profitability of a livestock-place is 11 % 2007, assuming herd size 130. The expected producer price changes reflect expected policy changes including the removal of production-based support. For details regarding Table 1, see [Uusitalo et al.2004]. Recent survey studies support pessimistic expectations regarding future profitability. In a deterministic continuous time model postponing investment is not optimal under decreasing income expectations [Dixit and Pindyck1994].

Markov Model

In Table 1 the states of future rate of return depend on the herd size and future producer price. Assume the possible Markov states are defined in terms of the future rate of return, corresponding to different scenarios regarding producer price change (for a given herd size). Denote the matrix of transition probabilities by $\mathbf{A}$. Let r_{it} denote the rate of return at time t in state i. The expected return $E(r_t)$ at time t is defined as:

$$E(r_t) = \sum_i P_{it} r_{it} \tag{4}$$

Table 1: Return on investment (ROI %) in milk production (2007 -10 % means 2007 ROI (including subsidy) when producer price decreases by 10 % from 2003 and investment subsidy increases by 20 % or 50 % depending on production unit

	ROI %	ROI %
	herd size 60	herd size 130
2003	24	30
2007 -10 %	10	16
2007 -12 %	7	14
2007 -15 %	4	11
2007 -17 %	2	8
2007 -20 %	0	5

where P_{it} is the probability that the rate of return is determined by state i at time t. The probabilities $\mathbf{P}_t = \{P_{it}\}$ associated with the different states r_{it} at time t are determined from:

$$\mathbf{P}'_t = \mathbf{P}'_0 \mathbf{A}^t, \tag{5}$$

where $\mathbf{P}_0$ denotes the vector of initial probabilities of the different return rates and corresponding subsidy levels.

In general, the transition probabilities at time t can be defined as function of the investment decision at time t. Formally, letting r_t denote the state at time t and I_t denote the investment decision at time t, the state transition probability is given as a function $P(r_t|r_{t-1}, I_t)$. A *Markov Decision Process* (MDP) is a Markov Model with the above modification, i.e. the transition probability matrix depends on the action taken in each stage [2]. For example, investment may increase productivity: this can be modelled by an MDP with a more advantageous transition matrix whenever investment takes place.

[2]Furthermore, in general the transition probabilities depend on the timing of the investment (e.g. due to fixed term investment subsidy programs).

Expected Value of Information

In stochastic programming literature [Birge and Louveaux1997], the *expected value of perfect information* measures the maximum amount a decision maker would be willing to pay for complete information:

Definition 1 *Let $f(x)$ denote the objective function to be maximized with respect to decision variable x. Let z denote a random variable. The expected value of perfect information (EVPI) can be measured as the difference [Birge and Louveaux1997]*

$$EVPI = E[\max\ f(x,z)] - \max\ E[f(x,z)]. \tag{6}$$

The first term in equation (6) corresponds to a *"wait-and-see" solution* and the second term to an *expected value maximizing solution.* EVPI can be used to measure the cost of imperfect information due to uncertain income and subsidies.

Before presenting numerical examples of the uncertainty cost, two optimization models of investment are introduced: a "wait-and-see" model and an expected value model, respectively.

Model 1: Investment in a Wait-and-See Model

In Model 1, like in [Dixit and Pindyck1994], the value of investment is observable at any time but the future values are random. The future value of investment is assumed to follow the same mean-reverting Markov process as the producer price does (cf. ibid.); with high probability, the value of investment remains unchanged. Specifically, at time t, the rate of return r_t is observed, and future return rate r_{t+1} is determined by a transition matrix modelling non-increasing income expectations (Table 2). The hypothetical Markov model as summarized in Table 2 is a simplified model of a mean-reverting income process, with a long run mean return rate 0.06. With high probability, the value of investment remains the same (except for the highest rate of return $r = 0.3$ associated with decreasing expectations).

Table 2: Transition Probabilities between States (ROI %)

	0.3	0.16	0.11	0.05	0
0.3	0.01	0.3	0.4	0.28	0.01
0.16	0.01	0.8	0.1	0.09	0
0.11	0.01	0.05	0.7	0.15	0.09
0.05	0.01	0.01	0.08	0.8	0.1
0	0	0	0.05	0.15	0.8

Formally, let y_t denote the value of investment in terms of income obtained when investing I_t at t, assuming infinite time horizon $T = \infty$:

$$y_t(I_t) = r_t I_t + E[\sum_{k=t+1}^{\infty} b^k r_k] I_t. \tag{7}$$

At each time $t = 1, ..., T$ the firm makes a decision on the level of investment I_t subject to constraint (2), with an aggregate budget I_a. The dynamic optimization problem subject to budget constraint (2) can be stated as

$$\max \; E[\sum_t b^t y_t(I_t) - b^t I_t] \tag{8}$$

Problem (8) subject to (1)-(2) can be solved recursively applying Bellman equation:

$$v(r_t) = \max_{I_t}\{(y_t - I_t) + bEv(r_{t+1})\}, \tag{9}$$

where $v(r_t)$ denotes the value function given state r_t.

In this paper, for simplicity, the investment model is formalized as an MDP as follows: Define an additional state $r_a = 0$ corresponding to a state where the budget has been used up. After investment has been made a new transition probability matrix applies: one where each state leads to state r_a with probability one.

Model 2: Expected Value Maximization

In Model 1, like in [Dixit and Pindyck1994], the value of investment at any time t is observable. In the case study summarized above, due to a high degree of policy uncertainty, the return rate r_t can be uncertain at the beginning of period t. Assuming r_t is observed at the end of period t, all terms affecting the value of investment are random. A risk-neutral decision maker in this case solves the Bellman equation:

$$v(E(r_t)) = \max_{I_t}\{E[y_t(r_t, I_t) - I_t] + bv[E(r_{t+1})]\}. \quad (10)$$

Using the terminology in [Keswani and Shackleton2006], the special case where the investment decision is made at the beginning of the time horizon corresponds to optimizing forward start net present value (NPV). According to formulation (10) the decision maker has the flexibility to make the investment decision at any time; the expected return can be determined based on the return observed previous time period. Assuming a stationary income process, however, there is no motivation for postponing investment; in this case NPV maximization is optimal.

4 Optimal Investment and Dynamic Cost of Uncertainty

In what follows, the uncertainty cost is studied applying the stochastic programming approach introduced above. First, a measure of dynamic (time-varying) uncertainty cost relative to expected investment is presented. This is based on a modification of Definition 1 to a dynamic context. Second, numerical examples are presented for the case study summarized above.

Consider the dynamic objective function $f(\{I_t\}, \{r_t\})$ defined as:

$$f(\{I_t\}, \{r_t\}) = \sum_t b^t y_t(r_t, I_t) - b^t I_t, \quad (11)$$

where y_t is formalized in (7), cf. problem (8). Directly applying Definition 1 for the expected value of perfect information (EVPI) to the dynamic objective (11), implies:

$$EVPI = E[\max_{\{I_t\}} f(\{I_t\}, \{r_t\})] - \max_{\{I_t\}} E[f(\{I_t\}, \{r_t\})]. \quad (12)$$

The first term on the right hand side in (12) corresponds to the expected value of the wait-and-see model (Model 1), based on assuming the value of investment is observable when the investment decision is made. The second term on the right hand side in (12) corresponds to maximizing the expected forward start NPV, i.e. to determining the optimal timing of investment at the beginning of the time horizon. Thus, the classical option value of postponing the investment decision [Dixit and Pindyck1994] can be seen as equivalent to EVPI in (12).

Assuming the investment decision can be made at any time even when the value of investment is not fully observable, EVPI can be modified to a dynamic uncertainty cost as follows. Let $\{I_t^*\}$ denote the solution to (9) (Model 1) and let $\{I_t^{**}\}$ denote the solution to (10) (Model 2). Applying expression (6) to the dynamic optimization problem (3), implies a *dynamic uncertainty cost*, EVPI(t), defined for period t as

$$EVPI(t) = E[y_t(I_t^*) - I_t^*] - E[E(y_t(I_t^{**})|r_{t-1}) - I_t^{**}] \tag{13}$$

where the first term corresponds to the expected value obtained at t when solving the wait-and-see model (Model 1) and the second term formalizes the corresponding expected value when the investment decision at time t is based on $E(r_t)$, given the observed return previous period (Model 2). Note that assuming a stationary income process, the second term in (13) is equivalent to maximizing expected NPV (postponing investment is not optimal); in this case dynamic EVPI(t) in (13) corresponds to a dynamic option value of postponing investment.

Consider the wait-and-see model, assuming the return from future investments is determined by transition probabilities in Table 2, where the different states are given in terms of different levels of return on investment, following the case study example. The net return when investing I_t at time t is given by expression (7). Letting $I_a = 10000$, $r_0 = 0.05$ and $b = 0.94$, problem (8) subject to (2) is solved numerically with backward recursion 10000 times, using Matlab [Fackler]. Figure 1 depicts the probability of investment, with mean 1 %. The investment probability decreases over time, reflecting decreasing income expectations.

Let $Pr_t(I)$ denote the probability of investment at time t when the state r_t is observed, and let $Pr_t'(I)$ denote the corresponding probability with expected value maximization (Model 2). A modification of EVPI (Definition 1) is to consider a cost measure relative the expected value of investment:

Definition 2 *A dynamic relative* EVPI, REPVI, *can be defined for time*

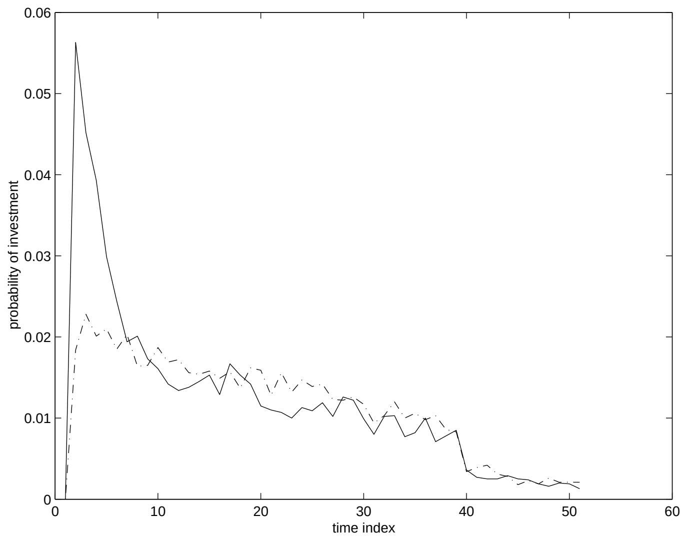

Figure 1: Probability of investment with $b = 0.94$, $r_0 = 0.05$ (dash-dotted curve), $r_0 = 0.11$ (solid curve)

period t as the weighted difference (cf. Definition 1):

$$REVPI(t) = \frac{E[y_t(I_t^*) - I_t^*]}{Pr_t(I_a)I_a} - \frac{E[E(y_t(I_t^{**})|r_{t-1}) - I_t^{**}]}{Pr_t'(I_a)I_a} \tag{14}$$

With a large number of iterations, the expected net value of the investment at time t, $E[y_t(I_t^*) - I_t^*]$ in (14), can be approximated by the mean net value. Figure 2 depicts REVPI(t) in the above example, assuming both expected net value terms in (14) are approximated by the corresponding mean net values over 150000 iterations. Assuming $b = 0.94$ the relative EVPI as defined in (14), when averaged over time, is 0.061. The outcome with $r_0 = 0.11$ is similar. The value of perfect information EVPI(t) is at least 5 % of total expenditure on investment. Increasing the stability of the value of investment e.g. by increasing the probability of no change in return to 0.95 (for all $r < 0.3$), reducing the probabilities of a change in return, implies a mean relative EVPI 0.046. On the other hand, if all the states are equally likely to follow from any given start state, the mean relative EVPI is approximately 0.07. Increasing the volatility of income thus increases the

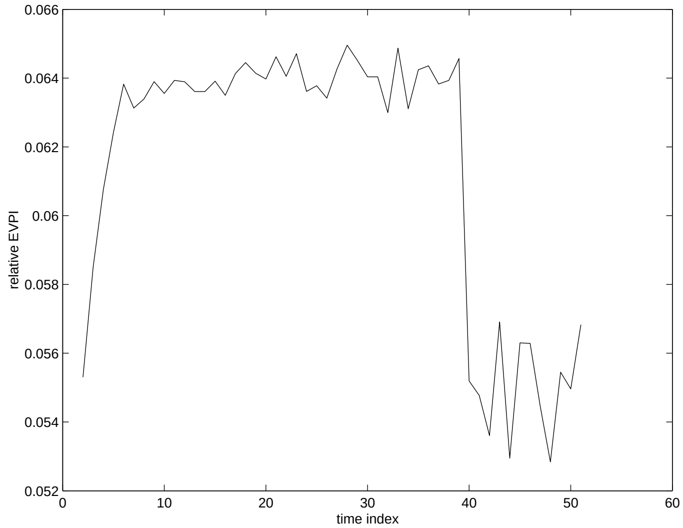

Figure 2: Relative EVPI, $b = 0.94$, $r_0 = 0.05$ (150000 runs)

cost of uncertainty. However, in this example, the mean uncertainty cost is quite robust to changes in underlying probabilities.

Dynamic Relative Option Value

The value of perfect information depends on the flexibility of decision-making in optimizing the expected value of investment. Let t^* denote the optimal timing of investment when maximizing expected forward start NPV at the beginning of the time horizon. A dynamic relative option value, O(t), can be defined as:

$$O(t) = \frac{E[y_t(I_t^*) - I_t^*]}{Pr_t(I_a)I_a} - \frac{\max\{\max_t b^t E(y_t - I_t), 0\}}{b^{t^*} I_a}. \qquad (15)$$

O(t) in (15) formalizes the dynamic option value relative to the expected value of investment. With $r_0 = 0.05$ the relative option value (not depicted) varies between 40 % and 42 % (the second term in (15) is zero). With $r_0 = 0.11$, keeping other parameters unchanged, the mean O(t) (not depicted) is 0.35, even if in this case the second term in (15) is positive.

In the special case where r_t has a stationary distribution, the two relative uncertainty cost measures, REVPI in (14) and the relative option value (15) are equivalent. Assuming a stationary distribution, the cost of imperfect information can be high. E.g. if $r_0 = 0.11$ and $b = 0.91$, assuming all states are equally likely, REVPI (and the relative option value) is on average more than 11 % of the expected investment.

Income Volatility and Uncertainty Cost

Increasing income volatility increases the cost of uncertainty. For example, assume the same transition probabilities as in Table 2 apply for states given in terms of ROI % defined for 3 time periods (instead of one period as above). This modification increases the variance between the states (ROI %). A motivation for this case is given by policy programs with fixed duration of 3 years. Assuming the same parameters as in Figure 2, this modification implies the mean REVPI over time is 17 % [3]. The mean investment probability is 2 %, twice the mean investment probability with the original definition of the transition probabilities (cf. Figure 1). In this case an increase in income volatility increases the investment probability.

The value of information is the higher the longer the time period with a certain income in the wait-and-see model, compared to expected value maximization with uncertain income. Consider a special case of stable income where the rate of return in the wait-and-see model remains at r_t for all future periods whenever investment is made at time t. For example, with $b = 0.94$, the mean REVPI in this case is approximately 72 % of the expected investment. With $b = 0.91$, average REVPI over time is more than 130 % of expected investment.

An Application to Policy Planning

Previous work based on a sector model of agriculture suggests that decoupling direct payments from production weakens the incentive for investment in dairy production and causes a temporary but significant slowdown in dairy investments [Lehtonen2004]. A key issue in planning an investment subsidy program is to ensure a target level of productivity-enhancing investments, despite decreasing expectations regarding future income. The return rate

[3] Further assuming the different states are equally likely, the average REVPI over time is more than 23 %.

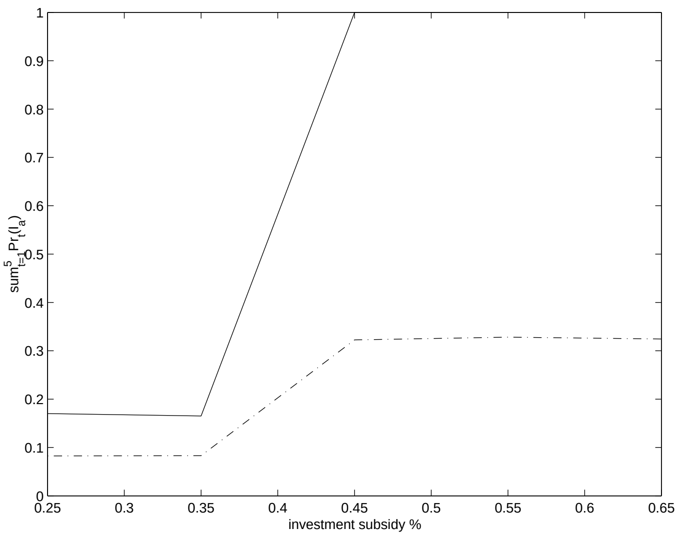

Figure 3: Cumulative investment probability as function of subsidy level, $b = 0.94$, $r_0 = 0.05$ (dash-dotted curve), $r_0 = 0.11$ (solid curve)

can be modelled as a Markov process that depends on the investment subsidy level. Assuming $b = 0.94$ in the wait-and-see model gives Figure 3, depicting the cumulative investment probability over the first 5 time periods as function of the investment subsidy (% of investment expenditure). E.g. with $r_0 = 0.05$, it can be observed that to affect investments, the subsidy must increase from its current level 35 % to 45 %: this more than triples cumulative investments during first five periods.

A high uncertainty cost deteriorates the efficiency of investment subsidies. For example, if the start state is $r_0 = 0.05$, and the investment subsidy is 0.35, the cumulative investment probability in the wait-and-see model with observable ROI is 8%, compared to 5% with expected value maximization with unobservable ROI.

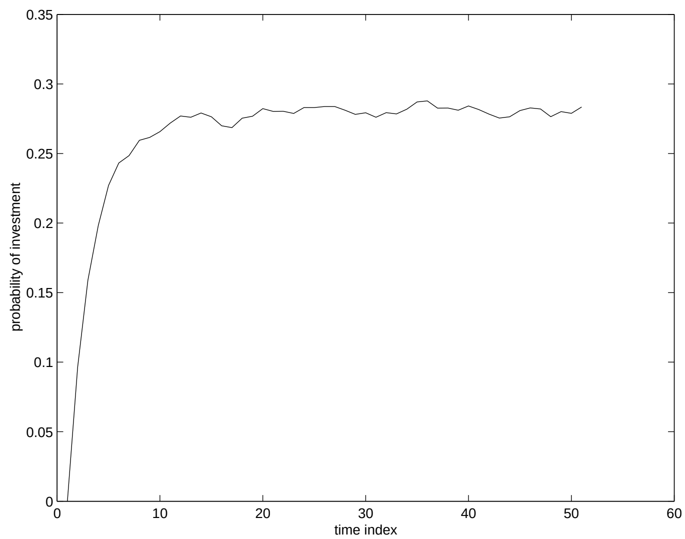

Figure 4: Probability of investment with $I = 200$, $b = 0.94$, $r_0 = 0.05$

Financial Constraints

Like in [Verkammen], assume now that the decision-maker decides at each time t on investment with period-specific financial constraints:

$$I_t \in \{0, I\},\ t = 1, ..., T. \tag{16}$$

With $I = 200$, $b = 0.94$ and $r_0 = 0.05$, the investment probability is depicted in Figure 4. Period-specific financial constraints may explain the postponement of part of the investments.

5 Investment by a Risk-Averse Agent

The above investment models are based on assuming the decision-maker is a risk neutral. To take risk explicitly into account, a modification of a standard MDP is presented, following [Levitt and Ben-Israel2001]. Examples suggest that the investment decision is sensitive to risk. Furthermore, the uncertainty-investment relation is nonlinear.

Risk in a Markov Decision Process

The idea that risk affects decision-making is not new in agricultural economics [Hardaker et al.1997]; A traditional approach can be summarized as follows. Consider a utility function in exponential form:

$$U(x) = 1 - e^{-\beta x}, \tag{17}$$

where β is a risk-aversion parameter. The expected value of utility (17) can be evaluated as [Hazell and Norton1986]

$$E(x) - \frac{\beta}{2} Var(x). \tag{18}$$

Stochastic programming [Prekopa1995] has been previously applied to decision-making in agriculture under uncertainty, see e.g. [Hazell and Norton1986]. A dynamic objective function accounting for risk is defined next based on a stochastic programming approach presented in [Levitt and Ben-Israel2001] (with applications to inventory control and the maintenance problem).

A Stochastic Programming Model

Definition 3 *The recourse certainty equivalent (RCE) of a scalar random variable* $\mathbf{Z}$ *is defined as*

$$S_U(\mathbf{Z}) = \sup_z \{z + EU(\mathbf{Z} - z)\}.$$

where U *is a concave function.*

Consider the quadratic utility function:

$$u(x) = x - \frac{\beta}{2} x^2, \tag{19}$$

where β is a risk parameter. Applying Definition 3 to utility function (19) gives the RCE associated with this utility:

$$S_\beta(\mathbf{X}) = E(\mathbf{X}) - \frac{\beta}{2} Var(\mathbf{X}) \tag{20}$$

where β is a risk parameter. An agent maximizing the criterion in (20) is risk averse if $\beta > 0$.

Definition 4 *The quadratic recourse certainty equivalent (RCE) of the random sequence* $\mathbf{X} = (X_1, ..., X_T)$ *is defined as [Levitt and Ben-Israel2001]*

$$S_{\beta_1,...,\beta_T}(\mathbf{X}) = \sum_{t=1}^{T} b^{t-1} S_{\beta_t}(\mathbf{X}_t) = \sum_{t=1}^{T} b^{t-1}\{E(\mathbf{X}_t) - \frac{\beta_t}{2} Var(\mathbf{X}_t)\} \tag{21}$$

where the β_t *parameters allow to model different risk attitudes in different stages. The "utility" obtained at time* t, $S_{\beta t}$ *is defined as the difference:*

$$E(\mathbf{X}_t) - \frac{\beta_t}{2} Var(\mathbf{X}_t). \tag{22}$$

The definition of the period-t RCE in equation (22) is analogous to RCE in equation (20). An alternative motivation for the definition of period t objective in equation (22) is given in equation (18).

The wait-and-see model (Model 1) can be modified to take risk into account, applying the utility model (22). This implies investment probabilities corresponding to maximizing quadratic recourse certainty equivalent (Definition 4). A numerical example is depicted in Figure 5, with $\beta = 10^{-4}$ [4], applying the exponential utility model in equation (17) (cf. (18) and (20)). In this example uncertainty lowers cumulative investment probability by almost 50%, compared to the case with observable value depicted in Figure 1 (with $r_0 = 0.05$). The investment probability depends on the amount of investment, dropping to zero at $I_a = 10300$. The relation between I_a and cumulative investment probability (not depicted) is nonlinear.

A positive uncertainty-investment relation was exemplified in section 4, when addressing the impact of increasing income volatility, assuming a risk-neutral decision maker. Taking risk into account in general modifies the uncertainty-investment relation.

Optimizing Forward Start NPV with Risk

In general, transition probabilities between different states of return on investment (cf. Table 1) depend on the timing on the investment. The proba-

[4]The Arrow-Pratt relative risk aversion (RRA) is defined at I_a as $-I_a U''(I_a)/U'(I_a)$. Using $\beta = 10^{-4}$ the RRA equals 1 at I_a; Arrow's conjecture that RRA approximately equals 1 is a common reference point. Recent empirical work considering the case of Turkish farmers [Binici et al.2003] suggests β varies between 0.04 and 0.49.

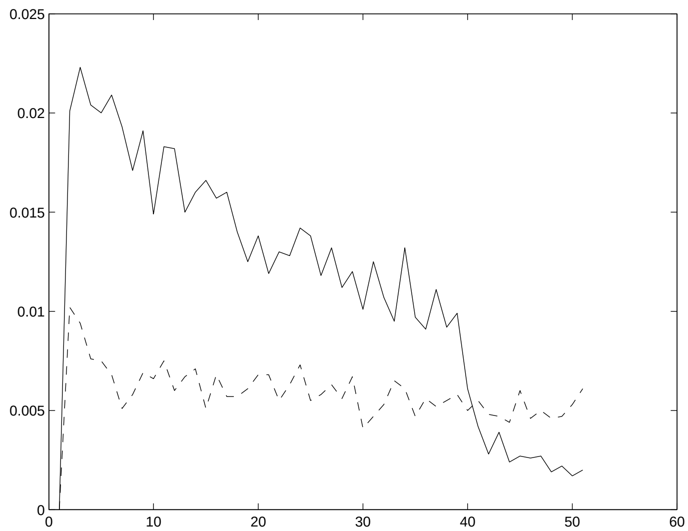

Figure 5: Investment probability ($b = 0.94$, $r_0 = 0.05$), first with risk-neutral firm as in Fig. 1 (upper curve), second assuming exponential utility with risk parameter $\beta = 10^{-4}$ (lower curve)

bilities may change e.g. due to a potential change in income and/or investment subsidies. Consider the special case of optimizing forward start NPV, assuming time-varying transition probabilities. Examples (not depicted) suggest that a time-varying variance can be a motivation for postponing the investment. However, the risk cost associated with valuable investments can be high enough to make the net utility negative, whereas risk does not affect the optimal timing of small investments at $t = 1$.

6 Conclusion

This paper has addressed the cost of income uncertainty in agricultural investment. Applying a stochastic programming approach, an operational formula for dynamic uncertainty cost is presented, modifying the classical expected value of perfect information. The cost of uncertainty depends on income volatility; in the special case of stationary income, the dynamic uncertainty cost is equivalent to a dynamic option value of postponing investment. Furthermore, numerical examples suggest that the investment decision is sensitive to risk. A cost associated with risk (variance) can be a source of an option value of postponing investment (along with period-specific financial constraints), even if the income process is non-increasing in time.

A topic for future work is a survey study of the subjective probability distributions. In some case study examples, the mean uncertainty cost is quite robust to changes in underlying probabilities. In future work, the model can be applied to e.g. studying the investment subsidy needed to maintain target investments under uncertainty. The efficiency of investment subsidy programs is deteriorated by the uncertainty regarding future income. The numerical optimization model presented in this paper is applicable to other dynamic resource allocation problems where the dynamic cost of uncertainty is relevant.

References

[Alvarez and Stenbacka2004] Alvarez, L. and R. Stenbacka: 2004, 'Optimal Risk Adaption: A Real Options Approach'. *Economic Theory* **23**, 123–147.

[Amegbeto and Featherstone1992] Amegbeto, K. and A. Featherstone: 1992, 'Risk Costs and the Choice of Market Return Index'. *Journal of Agricultural and Resource Economics* **17(1)**.

[Binici et al.2003] Binici, T., A. Koc, C. Zulauf, and A. Bayaner: 2003, 'Risk Attitudes of Farmers in Terms of Risk Aversion: A Case Study of Lower Seyhan Plain Farmers in Adana Province, Turkey'. *Turk J Agric For* **27**, 305–312.

[Birge and Louveaux1997] Birge, J. and F. Louveaux: 1997, *Introduction to Stochastic Programming*. Springer Series in Opereations Research.

[Bo and Lensink2005] Bo, H. and R. Lensink: 2005, 'Is the Investment-Uncertainty Relationship Nonlinear? An Empirical Analysis for the Netherlands'. *Economica* **72**, 307–331.

[Dixit and Pindyck1994] Dixit, A. and R. Pindyck: 1994, *Investment under Uncertainty*. Princeton: Princeton University Press.

[Fackler] Fackler, P., *Compecon Toolbox for Matlab*. http://www4.ncsu.edu/∼ pfackler/compecon/toolbox.html:.

[Hardaker et al.1997] Hardaker, J., R. Huirne, and J. Anderson: 1997, *Coping with Risk in Agriculture*. CAB International.

[Hazell and Norton1986] Hazell, P. and R. Norton: 1986, *Mathematical Programming for Economic Analysis in Agriculture*. MacMillian.

[Keswani and Shackleton2006] Keswani, A. and M. Shackleton: 2006, 'How real option disinvestment flexibility augments project NPV'. *European Journal of Operational Research* **168**, 240–252.

[Knapp and Olson1996] Knapp, K. and L. Olson: 1996, 'Dynamic resource managment: intertemporal substitution and risk aversion'. *American Journal of Agricultural Economics* **78**.

[Lagerkvist2005] Lagerkvist, C. J.: 2005, 'Agricultural Policy Uncertainty and Farm Level Adjustments: the Case of Direct Payments and Incentives for Farmland Investment'. *European Review of Agricultural Economics* **32(1)**, 1–23.

[Lehtonen2004] Lehtonen, H.: 2004, 'Impact of De-coupling Agricultural Support on Dairy Investment and Milk Production in Finland'. *Acta Agriculturae Scnadinavica Section C* pp. 46–62.

[Levitt and Ben-Israel2001] Levitt, S. and A. Ben-Israel: 2001, 'On Modelling Risk in Markov Decision Processes'. *in Optimization and Related Topics, A. Rubinov and B. Glover, editors* pp. 27–40.

[Prekopa1995] Prekopa, A.: 1995, *Stochastic Programming*. Kluwer.

[Uusitalo et al.2004] Uusitalo, P., A.-M. Heikkilä, and K. Pietola: 2004, 'The Effect of Decoupling CAP-subsidies on Profitability of Livestock Bilding Investments'. *In "CAP-uudistus Suomen maataludessa", H. Lehtonen ed. (in Finnish, attachments available upon request).*

[Verkammen] Verkammen, J., 'A Stochastic Programming Model of Direct Subsidy Payments and Agricultural Investment'. *Presentation at the Joint Annual Meetings of the American Agricultural Economics Association and Canadian Agricultural Economics Society. Montreal.*